# Nonlinear Observer and Robust Controller Design for Enhancement of Vehicle Lateral Stability

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This paper describes the design of a sliding mode controller to control wheel slip. A yaw motion controller (YMC), which uses a PID control method, is also proposed for controlling the brake pressure of the rear and inner wheels to enhance lateral stability. It induces the yaw rate to track the reference yaw rate, and it reduces a slip angle on a slippery road. A nonlinear observer is also developed to estimate the vehicle variables difficult to measure directly. The braking and steering performances of the anti-lock brake system (ABS) and YMC are evaluated for various driving conditions, including straight, J-turn, and sinusoidal maneuvers. The simulation results show that developed ABS reduces the stopping distance and increases the longitudinal stability. The observer estimates velocity, slip angle, and yaw rate very well. The results also reveal that the YMC improves vehicle lateral stability and controllability when various steering inputs are applied. In addition, the YMC enhances the vehicle safety on a split-$\mu$ road.

**Key Words :** Observer, Sliding Mode Control, PID (Proportional Integral Derivative) Control, YMC (Yaw Motion Controller), Side Slip Angle, Vehicle Stability

## Nomenclature

$a$ : Distance from cg to the front wheel
$A_w$ : Area of master cylinder
$b$ : Distance from cg to the rear wheel
$c_{af}, c_{ar}$ : Front and rear tire stiffness
$F_D$ : Drag force
$F_x$ : Longitudinal force
$F_y$ : Lateral force
$I_w$ : Rotating inertia of a wheel
$I_z$ : Vehicle moment of inertial around $z$ axis
$m$ : Vehicle mass
$P_b$ : Brake fluid pressure
$R_b$ : Distance from center of wheel to brake path
$R_w$ : Wheel radius
$t_f, t_r$ : Front and rear wheel distance
$T_{rolli}$ : Wheel torque due to resistance
$v_x$ : Longitudinal velocity
$\beta$ : Side slip angle
$\delta_f$ : Steering angle
$\lambda_d$ : Desired slip
$\mu$ : Friction coefficient
$\gamma$ : Yaw angle
$\theta$ : Rotational degree of wheel

## 1. Introduction

Since the introduction of the digitally controlled anti-lock brake system (ABS) in passenger cars in the late 1990s, electronic brake control systems have evolved rapidly and dynamically. A temporary climax in this process was the introduction of the electronic stability program (ESP), which is based on the well-known, proven ABS technology. ABS prevents wheel lockup during vehicle braking and maintains the capability of the tires

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to generate a lateral force. However, although it may contribute to stabilizing vehicle lateral motion, ABS cannot lead vehicle stability to the desired level (Fennel and Ding, 2000; Shibahara et al., 1993).

Therefore, many automotive manufacturers and researchers have concentrated on a vehicle stability control systems, such as ESP. The main task of ESP is to limit the slip angle in order to prevent vehicle spin. Another task of ESP is to keep the slip angle below the characteristic value to preserve some yaw moment gain. If the slip angle reaches the characteristic value, the gain will be low and the driver may notice that he or she starts to lose control of the vehicle and may start to panic. Therefore, ESP has to start controlling before this characteristic slip angle value is reached (Zanten, 2000).

Shibahara et al. (1993) proposed a method called the "$\beta$-method" for vehicle stability controller design and discussed how the yaw moment generated by the lateral force of the front and rear wheels changes in response to vehicle side slip. Kwak and Park (2001) designed a vehicle stability controller based on the multiple sliding mode control approach in order to overcome the uncertainty and nonlinear behavior of a real vehicle. Yi et al. (2003) developed an adaptive emergency braking controller using an observer-based dynamic friction model. Song and Boo (2004) also designed a sliding mode ABS controller and a PID steering angle controller to enhance lateral stability. However those controllers do not contain the observer to estimate slip or slip angle.

The goal of this paper was to design a sliding mode wheel slip controller that contains a nonlinear observer, and apply it to a nonlinear full-order vehicle model for lateral motion analysis. The other goal was to present a new proposed control technique to obtain greater lateral stability and to validate these two concepts through simulation.

## 2. Controller and Observer Design

The ABS and YMC controllers, and the observer developed in this study are based on a 15-degrees-of-freedom, nonlinear, full-car model (Fig. 1) (Song, 2005). However, the numerical model will not be described here. Detailed information on the vehicle model is given in Song and Boo (2004), and Song (2005).

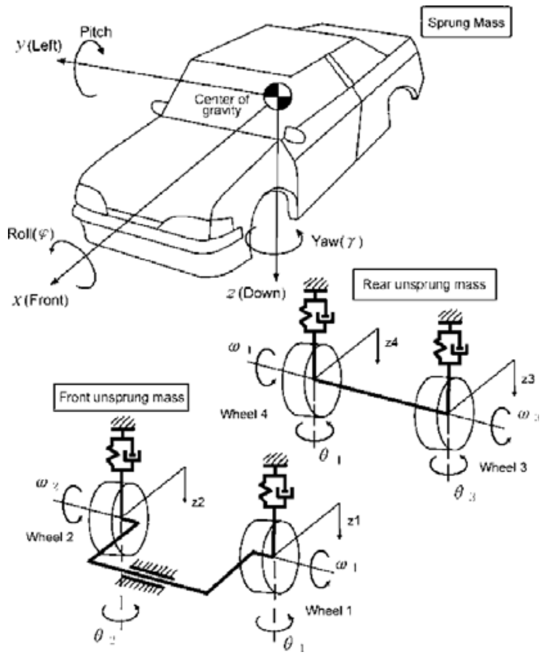


**Fig. 1** Fifteen degrees of freedom vehicle model

### 2.1 Sliding mode ABS slip controller

The stopping distance is thought to decrease as the braking torque increases. However, the stopping distance can also increase with increased braking torque if the wheels slide (Song, 2005). Therefore, a controller that maintains a suitable braking torque is required to ensure that the stopping distance is as short as possible. This study introduces a sliding mode controller to obtain the minimum stopping distance and to maintain steerability while braking.

From Song and Boo (2004), the wheel dynamics are

$$I_{wi}\dot{\theta}_i = -P_{bi}A_wR_b - F_{xi}R_w - T_{rolli} \quad (1)$$

$$\begin{aligned}\dot{\theta}_i &= -\frac{1}{I_{wi}}(P_{bi}A_wR_b + F_{xi}R_w + T_{rolli}) \\ &= -(K_iu + \tau_z + \tau_r)\end{aligned} \quad (2)$$

where $K_i = A_wR_b/I_{wi}$, $\tau_z = F_{xi}R_w/I_{wi}$, $\tau_r = T_{rolli}/I_{wi}$, and the control input $u_i = P_{bi}$. The dynamics of $\tau_x$ and $\tau_r$ are not known exactly, but they can be estimated as $\hat{\tau}_x$ and $\hat{\tau}_r$. The estimation error for $\tau_x$ and $\tau_r$ is assumed to be bounded by known values of $\tau_x^*$ and $\tau_r^*$.

In order for the slip of the braking system, $\lambda_{si}$, to track the desired slip, $\lambda_{di}$, the sliding surface is defined as

$$S=\left(\frac{d}{dt}+\lambda\right)\int_0^t \lambda_r dr \tag{3}$$

where $\lambda$ is a strictly positive constant and $\lambda_r=\lambda_{di}-\lambda_{si}$. The slip is defined during the braking $(v_x>\dot{\theta}\times R_w)$ as follows

$$\lambda_{si}=\frac{\dot{x}-\dot{\theta}R_w}{\dot{x}} \tag{4}$$

The derivative of the sliding surface is obtained from Equations (2), (3) and (4)

$$\dot{S}=\dot{\lambda}_r+\lambda\lambda_r=\frac{R_w}{\dot{x}^2}\Big[-(K_i u+\tau_z+\tau_r)\dot{x} -\dot{\theta}\ddot{x}+\frac{\dot{x}^2\lambda}{R_w}(\lambda_{di}-\lambda_{si})\Big] \tag{5}$$

The best approximation $\hat{u}$ of a continuous control law that gives $\dot{S}=0$ is

$$\hat{u}=-\frac{1}{\dot{x}K_i}\Big[(\hat{\tau}_x+\hat{\tau}_r)\dot{x}+\dot{\theta}\ddot{x}-\frac{\dot{x}^2\lambda}{R_w}(\lambda_{di}-\lambda_{si})\Big] \tag{6}$$

if we define

$$\bar{u}=\frac{\tau_r^*+\tau_x^*+\eta}{K_i}\operatorname{sgn}(S) \tag{7}$$

where $\eta$ is a strictly positive constant.

Since $u=\hat{u}+\bar{u}$, a control input $u$ can be determined as follows:

$$u=\hat{u}+\bar{u}=-\frac{1}{\dot{x}K_i}\Big[(\hat{\tau}_x+\hat{\tau}_r)\dot{x}+\dot{\theta}\ddot{x}-\frac{\dot{x}^2\lambda}{R_w}\Big] +\frac{\tau_r^*+\tau_x^*+\eta}{K_i}\operatorname{sgn}(S) \tag{8}$$

It satisfies the sliding condition, which maintains the scalar $S$ at zero (Slotine and Li, 1991),

$$\frac{1}{2}\frac{d}{dt}S^2=S\times\dot{S}\le-\eta|S|,\ (\eta\ge0) \tag{9}$$

The chattering problem caused by the control discontinuity of the sgn$(S)$ function can be eliminated by using a thin boundary layer of thickness $\Phi$ next to the switching surface. Hence, sgn$(S)$ can be replaced by the function sat$(S/\Phi)$ (Song and Boo, 2004).

### 2.2 Nonlinear observer design

In general, observers are implemented when certain state variables cannot be measured without unacceptable expense. If, as is the case here, the system has a nonlinear form, the direct application of a Luenberg observer is not possible (Kiencke and Nielsen, 2000). To overcome this problem, observer design using linearization is adopted.

The nonlinear observer estimates only those state variables that are essential for vehicle dynamic control: vehicle speed, $v_x$, the vehicle body side slip angle, $\beta$, and the yaw angle, $\gamma$. They can be described as follows (see Figs. 1 and 2):

$$\dot{v}_x=\frac{\cos\beta}{m_{total}}(Fx_1+F_{x2}+Fx_2+F_{x4}-F_D) +\frac{\sin\beta}{m_{total}}(Fy_1+Fy_2+Fy_3+Fy_4) \tag{10}$$

$$\dot{\beta}=\frac{\cos\beta}{m_{total}v_x}(Fy_1+Fy_2+Fy_3+Fy_4) -\frac{\sin\beta}{m_{total}v_x}(Fx_1+Fx_2+Fx_3+Fx_4-F_D)-\dot{\gamma} \tag{11}$$

$$I_z\ddot{\gamma}=aFy_1+\frac{t_f}{2}Fx_1+aFy_2-\frac{t_f}{2}Fx_2-bFy_3 +\frac{t_r}{2}Fx_3-bFy_4-\frac{t_r}{2}Fx_4 \tag{12}$$

and the input variables are

$$u=[Fx_1\ Fx_2\ Fx_3\ Fx_4\ \delta_f]^T$$

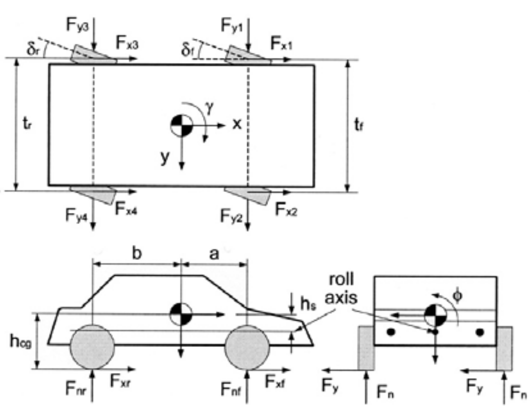


**Fig. 2** Vehicle model

To design a nonlinear observer, the following nonlinear system is used as the starting point

$$\begin{aligned}\dot{x} &= f(x, u) \\ y &= Cx\end{aligned} \tag{13}$$

The state observation follows

$$\begin{aligned}\dot{\hat{x}} &= f(\hat{x}, u) + L(\hat{x}, u) \times (y - \hat{y}) \\ \hat{y} &= C\hat{x}\end{aligned} \tag{14}$$

The observer gain matrix, $L$, must now be specified such that the estimation error, $\tilde{x}(t)$, tends to zero as $t \to \infty$. It must be found such that the observer dynamic matrix $F(\hat{x}, u)$ is constant and its eigenvalues lie to the left of the $j$-axis so that the solution, $\tilde{x}(t)$, of the estimation error differential equation tends to zero as $t \to \infty$ for any initial conditions. The suitable choice of $L(\hat{x}, u)$ is given by equating $F(\hat{x}, u)$ with a constant matrix $G$, whose values are predefined according to the desired dynamics.

$$F(\hat{x}, u) = \frac{\partial f}{\partial x}(\hat{x}, u) - L(\hat{x}, u) \times \frac{\partial c}{\partial x}(\hat{x}) \overset{!}{=} G \tag{15}$$

If we assume that the dynamic matrix $G$ is

$$G = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \tag{16}$$

The three eigenvalues $\lambda_1, \lambda_2$, and $\lambda_3$ must be chosen carefully in order to influence the dynamics of the observer gain matrix, $L$ (Kiencke and Nielsen, 2000). From Equation (15), the gain matrix, $L$, is

$$L(\hat{x}, u, \lambda_1, \lambda_3) = \begin{bmatrix} \frac{\partial \dot{v}_x}{\partial v_x} - \lambda_1 & \frac{\partial \dot{v}_x}{\partial \beta} & \frac{\partial \dot{v}_x}{\partial \dot{\gamma}} \\ \frac{\partial \dot{\beta}}{\partial v_x} & \frac{\partial \dot{\beta}}{\partial \beta} - \lambda_2 & \frac{\partial \dot{\beta}}{\partial \dot{r}} \\ \frac{\partial \ddot{\gamma}}{\partial v_x} & \frac{\partial \ddot{\gamma}}{\partial \beta} & \frac{\partial \ddot{r}}{\partial \dot{r}} - \lambda_3 \end{bmatrix} \tag{17}$$

### 2.3 Yaw moment control

The wheel slip controller was developed to prevent the vehicle from losing lateral stability while braking; however, it has several problems, which include not producing the yaw rate required for turning and causing the vehicle to spin. These problems make it difficult for the driver to maintain safe control of the vehicle (Song, 2005).

Previous studies with yaw rate feedback use the desired yaw rate as the reference model (Zanten, 2000; Song and Boo, 2004). The desired yaw rate corresponding to the steering angle and vehicle speed can be represented as follows:

$$\dot{\gamma}_{ref} = \frac{v_x \delta_f}{(a+b)\left[1 + \left(\frac{v_x}{v_{ch}}\right)^2\right]} \tag{18}$$

Where

$$v_{ch} = \frac{C_f C_r (a+b)^2}{m_{total}(C_f a - C_r b)}$$

Where, $m_{total}$ is the total mass of the vehicle.

When cornering and braking are applied simultaneously, the rear wheel on the inside of the turn dominates the vehicle direction (Fennel and Ding, 2000; Zanten 2000). Control of too many wheels might result in instability, so only the inner rear wheel is controlled, while the reference slips of the other three wheels remain unregulated by the YMC (Bang et al., 2001).

This control strategy can be embodied through the reference slip of the rear wheel of the inner side, which is calculated using a PID control method:

$$\begin{aligned}\lambda_{di} &= \lambda_{di} + K_P(\dot{\gamma}_{ref} - \dot{\gamma}) + K_d \frac{d}{dt}(\dot{\gamma}_{ref} - \dot{\gamma}) \\ &\quad + K_i \int (\dot{\gamma}_{ref} - \dot{\gamma})\, dt\end{aligned} \tag{19}$$

## 3. Simluation Results

### 3.1 Wheel slip controller evaluation

The straight maneuver is intended to characterize the basic vehicle model performance. Fig. 3 shows the numerical results on a wet asphalt road, where the target slip, $\lambda_{di}$, for the front and rear wheels is 0.2. The vehicle runs in straight lane and full brake pressure (30 bar) is applied at an initial speed of 30 m/s (108 km/hr). It is assumed that the friction coefficient, $\mu$, can be estimated approximately.

As shown in Fig. 3, when the vehicle is not equipped with the ABS and excessive brake pressure is applied, the wheel locks immediately, the wheel velocity drops to zero, and the slip becomes one (Equation (4)). It reduces longitudinal brake force, the stopping distance is increased, and the deceleration of the vehicle is reduced (Kiencke and Nielsen, 2000 ; Bang et al., 2001). Conversely, when the ABS is adopted, slip follows the target slip value and the vehicle velocity is reduced to about zero during 5 seconds after brake pressure is applied.

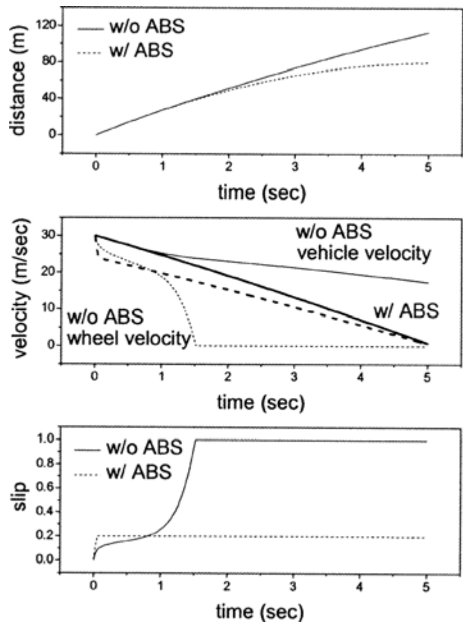


**Fig. 3** Performances of ABS on straight and wet asphalt

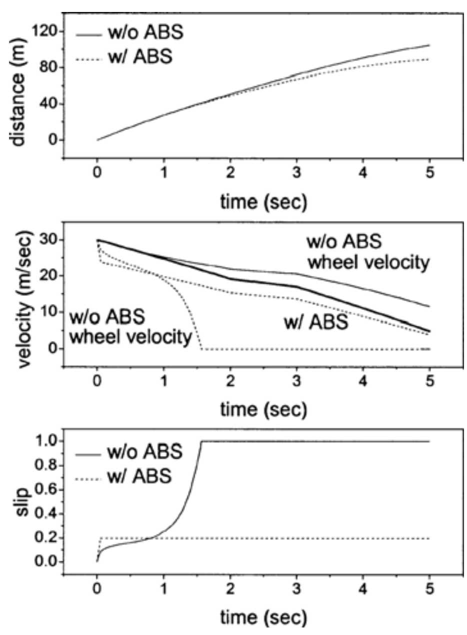


**Fig. 4** Performances of ABS on $\mu$ change road

The performance of the proposed ABS controller is also tested when the road condition is changed abruptly. Fig. 4 shows the simulation result on $\mu$ change road. The initial conditions are same with those of the previous study. A vehicle runs on wet asphalt road and passes through snow paved road during 2 to 3 seconds. The result shows that although the road condition is changed, the slip keeps the desired slip, which verifies the robustness of the ABS.

### 3.2 Observer evaluation

Figures 5, 6 and 7 show the performances of the observer. As examples, three drive situations are simulated in which the longitudinal and lateral dynamics are exited. From comparing the actual and observed state variables, such as vehicle velocity, slip angle, and yaw rate, the quality of the observer itself can be seen.

The first driving condition is that the driver applies ramp steering input, as shown in first plot of Fig. 5, and full brake input on wet asphalt. The actual and observed values of the velocity and yaw rate are similar, except for some initial spike-like noise. This noise can be removed easily using a low pass filter.

Figure 6 depicts the responses when a vehicle

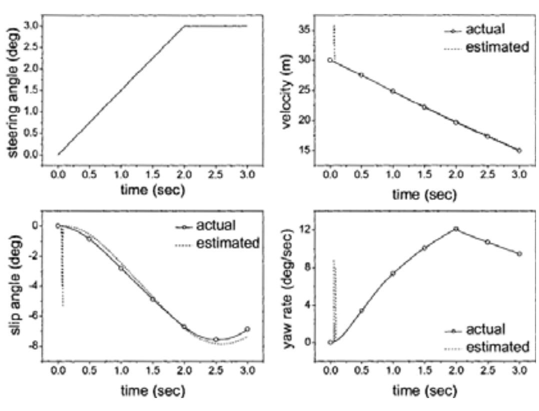


**Fig. 5** Performances of the observer when ramp steering input is applied on wet asphalt

is on snow paved road. Steer input and brake input are same with those of previous simulation. The observer shows satisfactory result even though the road condition is varied.

The responses when sinusoidal steering input and full brake input are applied simultaneously, and their observed values, are shown in Fig. 7. The initial velocity is 30 m/sec and the vehicle is on wet asphalt. The observed vehicle velocity and yaw rate are similar to the actual values.

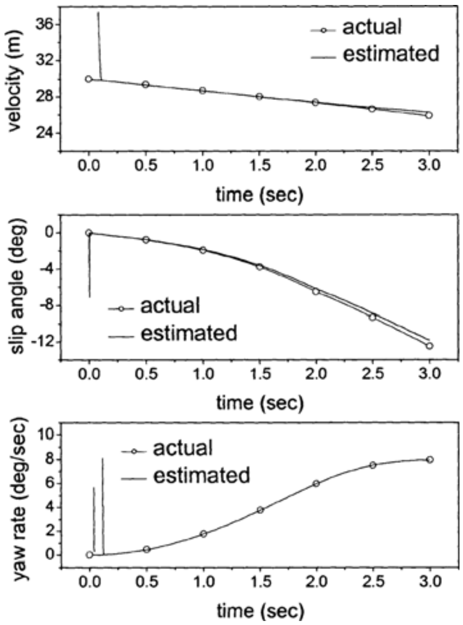


**Fig. 6** Performances of the observer when ramp steering input is applied on snow paved road

### 3.3 Yaw rate controller

There are three issues in the study of vehicle stability control systems : the observer design for estimating vehicle states, the vehicle stability controller design for lateral motion, and the actuator control for generating the yaw moment through the distribution of the brake force (Shibahara et al., 1993).

This study proposed a YMC (yaw motion controller) based on sliding mode control and a PID control to overcome the complexity of the control scheme, the nonlinear behavior of the vehicle, and the system uncertainty. The objective of the PID control is to control the brake pressure on the rear and inner wheels, and allow the yaw rate of the vehicle to track the reference yaw rate.

To evaluate the yaw rate controller, the steering input given in the first plot of Fig. 5 is applied again and the responses are examined. The initial speed is 30 m/s and the vehicle is on wet asphalt.

Figure 8 shows the responses of the vehicle, including the slip, slip angle, lateral acceleration, and yaw rate. The ABS controller forces the slip to stay near the desired value of 0.2, although the vehicle is in a turning maneuver. The YMC successfully makes the actual yaw rate coincide with the reference yaw rate. It also reduces the slip angle, and improves the stability and controllability during the turning maneuver.

Figure 9 shows the responses of the vehicle when brake input and sinusoidal steering input which are represented in first plot of Fig. 7 are

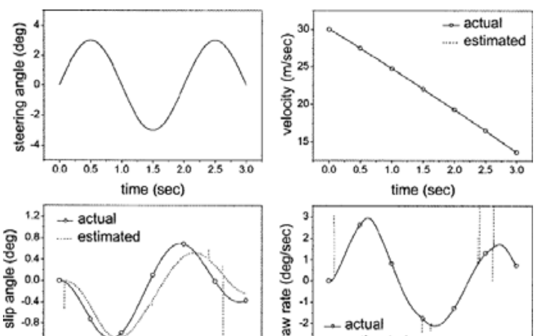


**Fig. 7** Performances of the observer when sinusoidal steering input is applied

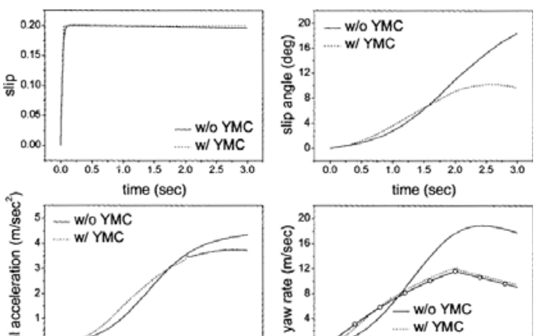


**Fig. 8** Performances of YMC when ramp steering input is applied

applied simultaneously. When sinusoidal input is applied, the vehicle will turn to the right and left alternately. However the slip remains at 0.2 due to robustness of the ABS controller and the yaw rate tracks the reference yaw rate very well because of YMC controller. The vehicle without YMC has a larger slip angle, which indicates that the controllability has deteriorated. In addition, the lateral acceleration is greater and the comportability and driveability becomes worse.

### 3.4 Split-$\mu$ road

When braking is applied on a split-$\mu$ road and the reference slip remains unchanged, the tires on the low-$\mu$ side produce smaller friction forces compared with the tires on the high-$\mu$ side, and the vehicle spins to the high-$\mu$ side suddenly, although an ABS is adopted (Bang et al., 2001). Therefore, on a split-$\mu$ road, the reference slip should be changed to make the friction forces of both tires similar and allow the vehicle to proceed in the forward direction.

In this study, when one wheel is on dry asphalt and the opposite wheel is on wet asphalt, the reference slips are established as 0.1 and 0.2, respectively. However, this compensation cannot induce the same friction forces and the vehicle will turn and yaw motion will be generated. However, when the YMC is applied, the yaw moment is balanced due to additional brake control and stable braking is possible.

Figure 10 shows the results of full braking on a

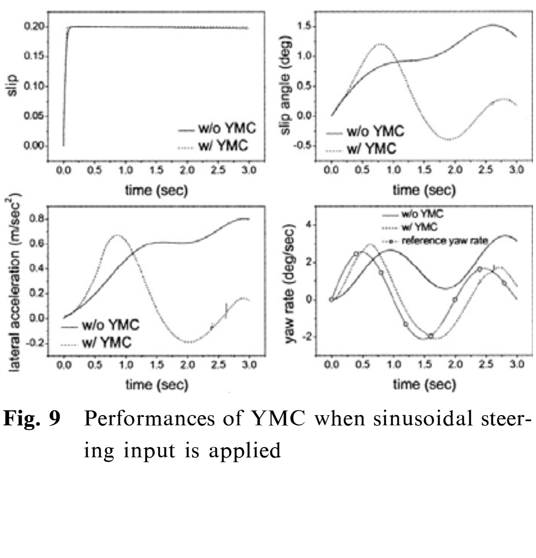


**Fig. 9** Performances of YMC when sinusoidal steering input is applied

split-$\mu$ road (left: wet asphalt; right: dry asphalt) with an initial speed of 30 m/sec. The steering input remains zero. When the ABS is applied, the yaw rate and lateral acceleration are reduced

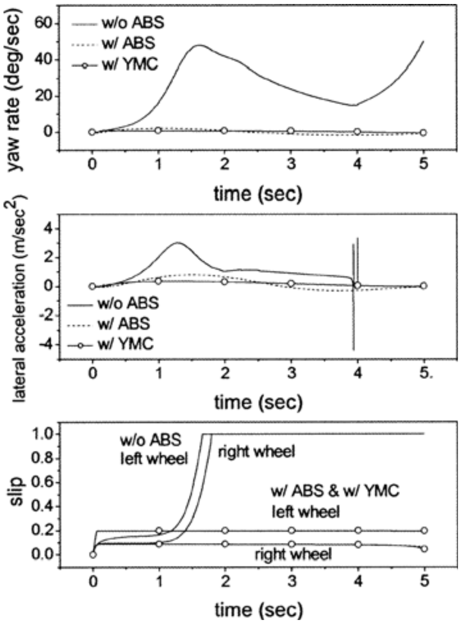


**Fig. 10** Performances of ABS and YMC on a split $\mu$ road

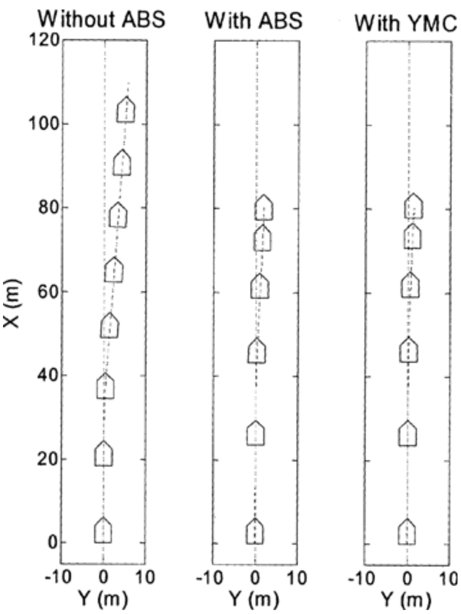


**Fig. 11** Trajectories comparison when a vehicle equips with and without ABS and YMC

drastically compared to those of without the ABS. When the YMC is used, their values remain near zero, which implies that the vehicle heads forward or changes its direction very slowly

Figure 11 represents the trajectories of vehicles without ABS, with ABS, and with YMC. Without the ABS, the vehicle shows the largest deviation from a straight lane, while with the YMC, the vehicle has the smallest deviation. The result also shows that the stopping distance is reduced by about 37% when the ABS or YMC is applied. These results prove that YMC enhances the longitudinal and lateral stability, and increases the vehicle safety on a split-$\mu$ road.

## 4. Conclusions

This simulation study investigated the performances of an ABS and YMC when a vehicle maneuvers under various conditions. The ABS is designed using a sliding mode control method, and the YMC is developed with the use of a PID controller. In order to perform this task, a nonlinear observer is also designed.

The conclusions from this study are as follows :

(1) When a vehicle runs on wet asphalt and excessive brake pressure is applied, the wheels lock and the braking distance is increased if the vehicle is not equipped with an ABS. The developed ABS controller works successfully because it prevents wheel lock, and the slip tracks the desired value very well when various steering inputs are applied.

(2) A nonlinear observer is designed and its performance is evaluated under various driving conditions. It estimates the vehicle velocity, vehicle body slip angle, and yaw rate, which all agree with the actual values very well.

(3) When a vehicle is in a braking and cornering maneuver, the YMC improves the lateral stability and controllability because it allows the yaw rate of the vehicle to track the reference yaw rate and reduces slip angle. The YMC also enhances the lateral stability and vehicle safety on a split-$\mu$ road.

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